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On the Systematic Proper Motions of Bright Stars relatively to Faint Stars in the Oxford Zones (+25° *to* +31°). By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In *Monthly Notices*, vol. lxiii. pp. 56–71, a preliminary discussion is given of the systematic proper motions in R.A. of bright stars relatively to faint, as determined from the Oxford photographs about 27° declination. The systematic proper motions were deduced by an *indirect* process as follows: Comparison of measures on plates taken at Oxford about 1894, with Cambridge meridian observations made about 1879, gives the sum of two quantities—viz. proper motion for fifteen years + Cambridge magnitude equation. Again, comparison of measures on plates taken at Oxford about 1899 with the Cambridge meridian observations gives

proper motion for 20 years + Cambridge magnitude equation.

Assuming the magnitude equation to be the same in both cases, we get by subtraction the systematic proper motions for five years.

2. Such a process has defects which are only too obvious and the results can only be put forward provisionally. I hope that before long we may undertake a direct determination of the relative proper motions in at least a part of our zone. We hope to finish the measures of our 1,180 plates prescribed in the programme before the end of the present year. When this is done it will be about ten years since the first of them was taken. We might therefore with advantage repeat the early plates, and measure the relative star places again after this interval of ten years. This proceeding could be restricted, in the first instance, to the odd zones +25°, +27°, +29°, +31°, so that we need only repeat four-sevenths of the work, and would be a general check on the previous measures, besides furnishing relative proper motions for ten years. As a general check it would not be much more laborious than the comparison of the measures on overlapping plates, for the numerical work necessary for this comparison, though straightforward, is considerable in amount. (See *Monthly Notices*, vol. lxi. p. 422.) As determining proper motions it would give greater completeness to the survey. The following remark is made by Herr E. Anding in reviewing Professor Newcomb's "Stars: a Study of the Universe" (*V. J. S. Ast. Ges.* 37. Jahr, iv. Heft, p. 333):—

"Schon S. 14 heisst es, der Bau des Universums sei das grösste Problem, das sich der Menschengeist jemals gestellt habe. Solchen Aeusserungen begegnet man nicht selten in der Literatur, häufiger in der letzten Zeit als früher. Doch hat Ref.

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noch niemals Gründe gefunden, warum diese 3*n* Anfangsconstanten wichtiger sein sollen, als die 3*n* Geschwindigkeiten. Nicht einmal ein heuristisches Moment lässt sich geltend machen: denn man wird die 3*n* Coordinaten gar nicht bestimmen können, ohne dass man zugleich die Geschwindigkeiten einführt."

To determine the places of a number of stars at any epoch is, after all, only half an investigation; we must also determine their proper motions, at least approximately, if we require their positions at other epochs.

3. Meanwhile it seems desirable to complete the former *indirect* investigation by treating the declinations in the same way as the R.A.'s, as promised in § 32 of the paper already quoted; and this is the object of the present paper. I shall first give for the *y* coordinate tables precisely similar to those formerly given for the *x* coordinate.

4. It seems unnecessary to repeat Tables I. and II., which will be found on p. 58 of vol. lxiii. Table I. gives the mean date of the groups and the number of plates in each. Briefly, there are about sixty "early" plates and sixty "late" plates in each of the zones +26°, +27°, +29°; and the mean difference of date is about five years. Table II. shows the numbers of stars of various magnitudes on these plates.

5. In Table III. we have what was called in the earlier paper the "raw material" for the discussion. The letters E and L are used to denote "early" and "late" plates.

TABLE III. (*y*.)

Mean y Residual (Oxford – Cambridge) for each group in units of 0″·3.

Zone +26°.

Octant of R.A. h h	Mag. 6·0. E.	Mag. 6·0. L.	Mag. 7·5. E.	Mag. 7·5. L.	Mag. 8·5. E.	Mag. 8·5. L.	Mag. 9·0. E.	Mag. 9·0. L.	Mag. 9·4. E.	Mag. 9·4. L.
0 – 3	− 5	− 4	−32	−17	−10	− 6	0	+ 4	+15	+ 7
3 – 6	− 9	+ 3	−30	− 9	+ 3	− 4	− 1	+ 4	+ 2	− 1
6 – 9	−53	+ 3	− 5	− 7	+ 2	− 1	0	− 1	+ 6	+ 5
9 – 12	−27	−17	−30	−19	+ 1	+ 8	− 4	− 9	+ 7	+ 3
12 – 15	− 7	− 3	−12	− 1	− 4	− 6	− 1	+ 5	+ 6	− 1
15 – 18	−40	0	+30	−16	+ 2	− 1	+13	0	− 8	+ 5
18 – 21	− 4	+ 4	− 4	− 5	0	− 2	− 1	− 3	+ 1	− 1
21 – 24	−13	−25	−22	− 7	− 4	−16	+ 2	+15	+ 6	+ 1
Mean	−19·8	4·9	13·1	−10·1	− 1·3	− 3·5	+ 1·0	+ 1·9	+ 4·4	+ 2·3

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Zone +27°.

Octant of R.A.	Mag. 6·0.		Mag. 7·5.		Mag. 8·5.		Mag. 9·0.		Mag 9·4	
h h	E.	L.	E.	L.	E.	L.	E.	L.	E.	L.
0− 3	*(−33)	−15	*(−25)	+7	*(−7)	−5	*(0)	+7	*(+13)	+12
3− 6	−12	+5	−13	−11	+1	−3	+3	0	+2	+6
6− 9	−32	−33	−9	−8	0	+1	+2	+3	+6	+12
9−12	−40	−15	0	−11	+5	−2	+18	+1	+10	+2
12−15	−9	−20	−8	−4	0	−6	−2	0	+10	+13
15−18	−13	+8	−3	−7	0	−3	+4	+2	+7	+3
18−21	+13	−11	0	−5	−5	−4	+2	−3	0	0
21−24	−7	+3	9	−13	0	−7	−1	+4	+11	+4
Mean	−16·6	−9·8	−8·4	−6·5	−0·8	−3·6	+3·3	+1·8	+7·4	+6·5

Zone +29°.

Octant of R.A.	Mag. 6·0.		Mag. 7·5.		Mag. 8·5.		Mag. 9·0.		Mag. 9·4.	
h h	E.	L.	E.	L.	E.	L.	E.	L.	E.	L.
0− 3	−60	−43	−19	−2	−4	−4	+1	+3	+11	+10
3− 6	−12	−9	−6	−2	−2	−9	+4	−4	+5	+12
6− 9	−6	−55	−4	−7	−7	−6	−1	0	+16	+7
9−12	−5	+5	−7	+5	+3	+2	−2	+1	+11	+2
12−15	−1	−37	−3	−10	−7	−2	−1	−7	+8	+43
15−18	+5	+19	+6	−25	−5	−3	0	+1	+7	+2
18−21	+1	+20	−1	−9	−5	−15	−3	−1	+2	−2
21−24	−18	−10	−9	−25	−4	−9	−3	−8	+6	+7
Mean	−12·0	−13·8	−5·4	−9·4	−3·9	−5·8	−0·6	−1·9	+8·3	+10·1

6. To follow the course of the previous paper we first set down as a preliminary result the differences between the means of the eight octants as they stand. These do not give us comparable proper motions, because the intervals, although all approximately five years, differ sensibly among themselves; but we get a sufficient test of the homogeneity of our material.

Table IV. (*y*.)

Approximate Mean P.M.'s in Decl. for five years in units of 0″·03.

	Magnitude.				
	6·0.	7·5.	8·5.	9·0.	9·4.
Decl. +26°	+14·9	+3·0	−2·2	+0·9	−2·1
„ +27	+6·8	+1·9	−2·8	−1·5	−0·9
„ +29	−1·8	−4·0	−1·9	−1·3	+1·8
Mean	+6·6	+0·3	−2·3	−0·6	−0·4

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* The mean of results for zones +26° and +29° has been substituted.

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7. These numbers differ in a marked manner from the corresponding numbers for R.A., in which there was a distinct progression. It seems worth repeating the former table for comparison :

TABLE IV. (*x*.)

Approximate Mean P.M.'s in R.A. for five years in units of 0″·03 *or* 0ˢ·00225.

	Magnitude.				
	6·0	7·5.	8·5.	9·0.	9·4.
Decl. + 26°	−18·6	−8·7	+1·5	−0·3	+2·6
„ +27°	− 6·9	−7·2	+0·2	+0·9	+2·5
„ +29°	−15·1	−4·8	−3·5	−1·1	+2·4
Mean	−13·5	−6·9	−0·6	−0·2	+2·5

In Table IV. (*x*) there was a sufficient accordance between the numbers for the three years to encourage us to believe that we were dealing with some systematic phenomenon ; but this accordance is not reproduced in Table IV. (*y*.) Indeed, the results for zone +26° and zone +29° are almost precisely at variance ; while zone +27° falls in between the two, as is shown by its agreement with the means. We are driven to infer that the systematic proper motions, in declination at any rate, are very small compared with those which do not follow any systematic law of drift ; or possibly that the accidental errors of measurement are larger than we should naturally expect. And in the light of these new results we naturally regard the apparent accordance of Table IV. (*x*) with more caution.

8. We proceed nevertheless to combine the three zones. We may still hope to diminish the "accidental errors" by so doing, though the whole work is put on a more tentative basis. Table VI. (*y*) has been formed in the same way as Table VI. of the former paper. We may first repeat Table V., as it is required for the formation of Table VII. It shows the number of plates for each octant of R.A. and the mean interval in years.

TABLE

Mean Results

(The unit

Octant of R.A.	Interval in Years.	6·0			7·5		
		E.	L.	L—E.	E.	L.	L—E.
h h							
0− 3	5·2	−41·7	−27·3	+14·4	−23·3	− 5·6	+17·7
3− 6	5·2	−11·3	− 0·8	+10·5	−15·2	− 7·1	+ 8·1
6− 9	4·9	−27·6	−26·8	+ 0·8	− 6·8	− 7·3	− 0·5
9−12	5·3	−16·2	− 8·6	+ 7·6	−15·3	− 7·4	+ 7·9
12−15	4·5	− 3·6	−13·3	− 9·7	− 5·9	− 3·1	+ 2·8
15−18	4·6	− 4·5	+ 7·8	+12·3	+ 6·2	−14·2	− 20·4
18−21	4·1	+ 1·4	+ 3·6	+ 2·2	− 1·9	− 5·9	− 4·0
21−24	4·0	−14·1	− 1·5	+12·6	−13·1	−14·5	− 1·4

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Table V.

Octant of R.A.	0h–3h	3h–6h	6h–9h	9h–12h	12h–15h	15h–18h	18h–21h	21h–24h
Num. E plates	12	23	29	13	29	19	33	38
Num. L. plates	16	27	24	38	23	35	23	12
Mean interval	5·2	5·2	4·9	5·3	4·5	4·6	4·1	4·0

Table VII. (*y*.)

Deduced Relative Motions in each Octant.

(The unit is 0″·03.)

Octant.	Mag. 6·0.	Mag. 7·5.	Mag. 8·5.	Mag. 9·0.	Mag. 9·4.	Gradients. Bright Stars.	Gradients. Mean.	Gradients. Faint Stars.
h h								
0 − 3	+2·8	+3·4	+0·2	+0·6	−0·6	−0·4	+1·4	+0·9
3 − 6	+2·0	+1·6	−1·2	−0·5	+0·7	+0·3	+1·0	−2·1
6 − 9	+0·2	−0·1	−0·1	0·0	−0·3	+0·2	+0·1	+0·2
9 − 12	+1·4	+1·5	−0·1	0·0	−1·4	−0·1	+0·9	+1·4
12 − 15	−2·2	+0·6	−0·1	+0·6	+0·2	−1·9	−0·5	−0·3
15 − 18	+2·7	−4·4	+0·1	−0·3	−0·4	+4·7	−0·3	+0·6
18 − 21	+0·5	−1·0	−0·5	−0·3	−0·6	+1·0	+0·1	+0·1
21 − 24	+3·2	−0·4	−1·2	+1·0	−0·7	+2·4	+0·8	−0·6
Mean	+1·33	+0·15	−0·36	+0·14	−0·39	+0·78	+0·44	+0·03

9. The differences between early and late plates have then been divided by the mean intervals shown in Table V., so as to get the annual P.M.'s., shown in Table VII., still expressed in units of 0″·03. The "gradients" in the last three columns have been formed as follows: Subtracting from the mean of the results for magnitudes 6·0 and 7·5 the mean of the three results for 8·5, 9·0, and 9·4, and dividing the difference by 2·2 (the difference of the mean magnitudes), we get the mean relative P.M. for one magnitude, which is called the mean gradient. But

VI. (*y*.)

for all Zones.

is 0″·03.)

8·5			9·0			9·4		
E.	L.	L—E.	E	L.	L—E.	E.	L.	L—E.
−6·0	−4·8	+1·2	+0·7	+3·8	+3·1	+12·3	+9·3	−3·0
+0·8	−5·5	−6·3	+2·3	−0·3	−2·6	+ 2·7	+6·1	+3·4
−1·8	−2·1	−0·3	+0·7	+0·5	−0·2	+ 9·1	+7·7	−1·4
+2·4	+1·7	−0·7	−1·2	−1·4	−0·2	+ 9·4	+2·2	−7·2
−5·3	−5·7	−0·4	−1·1	+1·8	+2·9	+ 7·8	+8·9	+1·1
−2·9	−2·3	+0·6	+2·4	+1·1	−1·3	+ 5·4	+3·5	−1·9
−3·2	−5·3	−2·1	−1·4	−2·6	−1·2	+ 1·3	−1·0	−2·3
−3·2	−8·1	−4·9	−1·0	+2·9	+3·9	+ 7·1	+4·3	−2·8

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as the gradient may not be uniform two other columns have been formed, that for "bright" stars by comparing the results for 6·0 and 7·5 and dividing by 1·5, and that for "faint" stars by comparing 8·5 and 9·4 and dividing by 0·9. The accidental errors of these columns are, of course, much larger than that for the mean.

10. The question raised in the previous paper was: How far can these relative proper motions of bright and faint stars be referred to the Sun's motion through space? The expressions for the parallactic motion are

$$\mu_\alpha = +(\text{V} \cos \text{D} \sec \delta) \times \varpi \times \sin(\alpha - \text{A})$$

$$\mu_\delta = +(\text{V} \cos \text{D} \sec \delta) \times \varpi \times \text{K},$$

where

$$\text{K} = \sin \delta \cos \delta \cos(\alpha - \text{A}) - \cos^2 \delta \tan \text{D},$$

V being the Sun's velocity, A and D the R.A. and Decl. of the point towards which the Sun is moving, and ϖ the annual parallax of a star whose R.A. and Decl. are α and δ.

11. Assuming V cos D sec δ to be constant for the results now under discussion, the values of (V cos D sec δ) × ϖ were found in the previous paper (p. 68) by dividing μ_α by sin $(\alpha - \text{A})$. They can now be found independently from the P.M.'s in declination by dividing μ_δ by K, and a comparison of the results will tell us how far we are right in ascribing these relative P.M.'s to the Sun's motion in space. The comparison is shown in Table VIII.

TABLE VIII.

Octant. h h	μ_α	sin $(\alpha - \text{A})$.	$\mu_\alpha/\sin \alpha - \text{A}$	μ_δ.	K.	μ_δ/K.
0 − 3	−2·1	+·9	−2·3	+1·4	−·6	−2·3
3 − 6	−0·1	+·4	−0·2	+1·0	−·8	−1·3
6 − 9	−0·8	−·4	+2·0	+0·1	−·8	−0·1
9 − 12	−0·1	−·9	+0·1	+0·9	−·6	−1·5
12 − 15	−0·7	−·9	+0·8	−0·5	−·3	+1·7
15 − 18	+0·5	−·4	−1·1	−0·3	(−·1)	(+3·0)
18 − 21	−0·3*	+·4	−0·8*	+0·1	(−·1)	(−1·0)
21 − 24	−0·8	+·9	−0·9	+0·8	−·3	−2·7

12. The values of sin $(\alpha - \text{A})$ and K are only approximate, and for the groups $15^{\text{h}} - 18^{\text{h}}$ and $18^{\text{h}} - 21^{\text{h}}$ the value of K cannot be trusted as a divisor. (An accurate value might just possibly have a small positive sign.) If we exclude these groups and arrange the others according to the value of $\mu_\alpha/\sin(\alpha - \text{A})$ we get the following quantities, which should agree :—

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* There is unfortunately a slight error in Table VII. of the previous paper, which is here corrected. The consequent errata are given at end of this paper.

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Octant.	0^h-8^h	21^h-24^h	3^h-6^h	9^h-12^h	12^h-15^h	6^h-9^h
$\mu_\alpha/\sin(\alpha-A)$	−2·3	−0·9	−0·2	+0·1	+0·8	+2·0
μ_δ/K ...	−2·3	−2·7	−1·3	−1·5	+1·7	−0·1

13. Considering the large accidental error which obviously affects the results, the accordance is not unsatisfactory. If we accept it as showing that we are getting some indication, however rough, of the relative parallaxes of bright and faint stars in the different octants, then we may combine the two sets of results to improve this determination. Let us change the sign of μ_α when $\sin(\alpha - A)$ is negative, and of μ_δ when K is negative, and add the two together, dividing by the numerical sum of the factors, as in Table IX. There is now less reason for excluding the groups bracketed.

Table IX.

Octant. h h	$\pm\mu_\alpha\pm\mu_\delta$.	$\pm\sin(\alpha-A)\pm K$.	Ratio $=\varpi V$ $\cos D \sec \delta$.	Galactic Latitude. °
0 − 3	−3·5	1·5	−2·3	−34
3 − 6	−1·1	1·2	−0·9	−13
6 − 9	+0·7	1·2	+0·6	+22
9 − 12	−0·8	1·5	−0·5	+61
12 − 15	+1·2	1·2	+1·0	+80
15 − 18	−0·2	0·5	−0·4	+40
18 − 21	−0·4	0·5	−0·8	+ 2
21 − 24	−1·6	1·2	−1·3	−25

14. The approximate galactic latitude of the group has been inserted, as in Table XI. of the former paper (*Monthly Notices*, lxiii. p. 68). If we arrange the results simply according to latitude we get the following series, the results of the former paper, obtained from R.A. alone, being added for comparison :—

Deduced Value of ϖ V *cos* D *sec* δ *for different Galactic Latitudes.*

Gal. Lat.	+2°	−13°	+22°	−25°	−34°	+40°	+61°	+80°
R.A. and Decl.	−0·8	−0·9	+0·6	−1·3	−2·3	−0·4	−0·5	+1·0
R.A. alone	−0·8	−0·2	+2·0	−0·9	−2·3	−1·1	+0·1	+0·8

15. It does not, therefore, seem impossible that there may be some reversal, over a limited range of magnitudes, of the law that parallax and brightness diminish together, which is specially marked at about 35° from the galaxy. At any rate a case has been made out for further inquiry.

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16. We will now consider the results obtained from another point of view, which depends, however, on an assumption which may not be true.

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Can we assume that the Cambridge magnitude equation is the same for all R.A.'s?

This assumption has usually been made, but it must at any rate be carefully examined, and recent work (see *Obs.* xxvi. p. 210) seems to show that it is sometimes, perhaps always, erroneous. But by adopting it for an independent investigation of the quantities under discussion we shall throw some light on the validity of the assumption itself. The use to be made of it will now be stated.

17. Consider for simplicity two stars only, a bright B and a faint F. About 1879 these two stars were observed at Cambridge, and taking B as correct, the observed R.A. of F was too large, by M, say. About 1894 these stars were photographed at Oxford. F had meanwhile been moving with reference to B and had increased its relative R.A. by 15P, say, where P is the relative P.M. per year. Hence we should find, on comparing the Oxford photograph with the Cambridge places,

$$M_1 = M + 15P,$$

as the Cambridge magnitude equation, instead of M, as before. Again, other photographs were taken at Oxford about 1899. Had they been of the same actual stars we could have compared them directly with those of 1894, and avoided using the Cambridge observations as an intermediary; but they were of different regions, and we thus find for the Cambridge magnitude equation

$$M_2 = M' + 20P'.$$

Now in what precedes it has been assumed that in the same part of the sky (*i.e.* in each separate octant) $M = M'$ and $P = P'$, for the mean of a large number of stars. The former assumption is probably correct; but the latter is more doubtful, as we noticed in § 7 of this paper. Still we have hitherto been working on this assumption; and from it we have deduced, by subtracting the above equations,

$$5P = M_2 - M_1,$$

and if we substitute this value of P in either of the above equations we get

$$M = 4M_1 - 3M_2.$$

18. Now, if we could assume that the value of M should be the same for all R.A.'s we could get for it by combining all the octants a much better value, M_0, than the value deducible from a single octant alone. Using this mean value we could then find P for each octant from the equations

$$35P = (M_1 + M_2) - 2M_0 \quad \dots \quad \dots \quad (a)$$

and a comparison of the results with those already found from the equations

$$5P = M_2 - M_1 \quad \dots \quad \dots \quad \dots \quad (b)$$

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would give a valuable check upon them. The same would, of course, hold good if we in any way knew the value of M_0 in equation (a) *independently* of the present observations. But to substitute the value determined from the present observations, viz.

$$M = 4M_1 - 3M_2 \quad \dots \quad \dots \quad \dots \quad (c)$$

simply reduces equation (a) to identity with (b).

19. Hence we cannot get any value for the P.M.'s independent of that already found unless we can find some value for M different from that directly furnished by equation (c); though if M may be regarded as the same for all octants, its mean value, derived from the mean of eight equations like (c), would practically satisfy requirements. Let us in the first place see whether it seems likely that M *is* constant in value; and first consider the results for the y coordinate, collected in the present paper.

20. Recurring to Table VI. (y), the letters E and L correspond to what we have called M_1 and M_2. Now from (c)

$$\begin{aligned} M &= M_2 - 4(M_2 - M_1) \\ &= L - 4(L - E). \end{aligned}$$

We can thus form M very readily by subtracting four times the column (L − E) from the column preceding it. The factor four should not be absolutely constant: it is really the ratio of the interval between the Cambridge observations and the Oxford L plates to the interval between L and E. The former may be taken as 20; the latter is given in Table VI. The results are shown in Table X. (y) for each magnitude; and in the column "Mean Gradient" a result is found, as before, by subtracting the mean of the last three columns from the mean for 6·0 and 7·5, and dividing by 2·2.

TABLE X. (y.)

Values of M *deduced from Table VI.* (y) *in units of* 0″·03.

Octant of R.A.	Factor.	6·0	7·5	8·5	9·0	9·4	Mean Gradient.
h h							
0 − 3	3·8	−82·0	−72·9	− 9·4	− 8·0	+20·7	−35·7
3 − 6	3·8	−40·7	−37·9	+18·4	+ 9·6	− 6·8	−21·0
6 − 9	4·1	−30·1	− 5·2	− 0·9	+ 1·3	+13·4	−10·0
9 − 12	3·8	−37·5	−37·4	+ 4·4	− 0·6	+29·6	−22·0
12 − 15	4·4	+29·4	−15·4	− 4·0	−11·0	+ 4·1	+ 4·8
15 − 18	4·4	−46·3	+75·6	− 4·9	+ 6·8	+11·9	+ 4·5
18 − 21	4·9	− 7·2	+13·7	+ 5·0	+ 3·3	+10·3	− 1·3
21 − 24	5·0	−64·5	− 7·5	+16·4	−16·6	+18·3	−19·1
Mean	4·3	−34·9	−10·9	+ 3·1	− 1·9	+12·7	−12·5

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21. Now a mere glance at this table is sufficient to show that the values of M indicated are far too large and irregular. Take, for instance, the column headed "Mean Gradient," which represents the mean magnitude equation *per magnitude* in declination at the epoch 1875·0, if the systematic P.M.'s we are dealing with are treated as accurate and carried back twenty years; and its value in the first octant is found to be $-35·7 \times 0''·03 = -1''·07$, while in the 5th and 6th it is about $+0''·14$. This range is quite inadmissible. Even the mean value $-12·5$ or $-0''·37$ is too large, being more than double the quantity found, for instance, by Auwers (*Ast. Nach.* No. 3844, p. 72).

22. It would, therefore, appear that our results do not truly represent *systematic* proper motions. Either the individual proper motions are too large compared with any systematic drift to satisfactorily nullify one another in our means or there is some unknown source of accidental error. It may be that the magnitude equation varies with R.A., but it certainly does not vary as much as would be indicated by Table X. (*y*), and we have no independent determination of its variation.

23. But it is still of interest to find what systematic P.M.'s would be given by the assumption that M_0 was the same for all R.A.'s, from the equation (*a*) of § 18. In this method we practically cease to differentiate E and L plates. We take the mean of both, and find how much M has strayed from its original value M_0 in each octant owing to systematic P.M. during the interval of about 17·5 years between the Cambridge observations and the Oxford plates.

24. We must, however, decide what value to use for M_0. In Auwers's paper above referred to (*Ast. Nach.* No. 3844, p. 72) the values of M_0 referred to magnitude 9·0 as zero, and expressed in our units of $0''·03$, would be

TABLE XI. (*y*.)

Values of M_0.

	6·0	7·5	8·5	9·0	9·4	Mean Grad.
Auwers	−12·3	−9·7	−4·0	0·0	+ 3·2	− 4·9
Table X. (*y*) (Means)	−33·0	−9·0	+5·0	0·0	+14·6	−12·5

In order to render the new results as much as possible independent of the old we will adopt the results given by Auwers as they stand, and find the values of P for each octant from equation (*a*) of § 18, where for M_1 and M_2 we take simply the E and L of Table VI. (*y*.) The results are given in Table XII. (*y*), and the results of Table VII. (*y*) are added for comparison.

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TABLE XII. (y.)

Systematic P.M.'s in Declination deduced from the 17½ years' interval between Cambridge Meridian Observations and Oxford Plates, assuming Auwers's determination of the Magnitude Equation to hold good in all R.A.'s.

Octant of R.A.	Mag. 6·0.		Mag. 7·5.		Mag. 8·5.		Mag. 9·0.		Mag 9·4.		Mean Gradient.	
	New Det.	Old (Table VII.)	New	Old	New	Old	New	Old	New	Old	New Det.	Old (Table VII.)
h h												
0– 3	−1·3	+2·8	−0·3	+3·4	−0·1	+0·2	+0·1	+0·6	+0·4	−0·6	−0·4	+1·4
3– 6	+0·4	+2·0	−0·1	+1·6	+0·1	−1·2	+0·1	−0·5	+0·1	+0·7	0·0	+1·0
6– 9	−0·9	+0·2	+0·2	−0·1	+0·1	−0·1	0·0	0·0	+0·3	−0·3	−0·2	+0·1
9–12	0·0	+1·4	−0·1	+1·5	+0·3	−0·1	−0·1	0·0	+0·2	−1·4	−0·1	+0·9
12–15	+0·2	−2·2	+0·3	+0·6	−0·1	−0·1	0·0	+0·6	+0·3	+0·2	+0·1	−0·5
15–18	+0·8	+2·7	+0·3	−4·4	+0·1	+0·1	+0·1	−0·3	+0·1	−0·4	+0·2	−0·3
18–21	+0·8	+0·5	+0·3	−1·0	0·0	−0·5	−0·1	−0·3	−0·2	−0·6	+0·3	+0·1
21–24	+0·3	+3·2	−0·2	−0·4	−0·1	−1·2	+0·1	+1·0	+0·1	−0·7	0·0	+0·8

25. The last pair of columns alone is sufficient to show that there is no correspondence between the new determination of systematic proper motions and those of Table VII.: either one or the other set is quite untrustworthy. We can improve the accordance a little by assuming a different value for the original magnitude equation of the Cambridge observations, which means simply adding a constant to the column headed "New Determination;" but the improvement is not sufficient to give at length.

26. Before proceeding to examine what values the new determination would give to the relative parallax of bright and faint stars, as in Table VIII., we will form the corresponding results for R.A. on the new plan. The following tables, X. (x), XI. (x), and XII. (x), correspond precisely to those given above for (y), and it is perhaps unnecessary to repeat the explanations. They are derived from Table VI. on p. 62 of *Monthly Notices*, vol. lxiii., in exactly the same way as the tables for (y) are derived from Table VI. (y) of the present paper.

TABLE X. (x).

Values of M *deduced from Table VI. ("Monthly Notices," vol. lxiii. p.* 62) *in units of* 0″·03.

Octant of R.A.	Factor.	6·0	7·5	8·5	9·0	9·4	Mean Gradient.
h h							
0– 3	3·8	+165·7	+146·7	+57·1	+34·8	−45·0	+63·9
3– 6	3·8	+ 25·5	+ 13·0	−14·1	−11·3	−34·1	+17·8
6– 9	4·1	+ 21·6	+ 76·2	+ 1·6	−16·0	−63·7	+34·1
9–12	3·8	+ 74·3	− 0·7	+23·9	−19·3	−22·6	+19·5
12–15	4·4	+ 53·6	+ 70·1	+22·0	+12·7	−61·4	+32·2
15–18	4·4	+ 21·3	+ 17·0	+ 3·5	−27·9	+ 5·1	+11·7
18–21	4·9	+ 68·4	+ 48·4	+46·7	−11·2	−20·7	+24·3
21–24	5·0	+103·8	+ 51·8	+13·9	+19·6	−34·3	+35·5
Mean	...	+ 66·8	+ 52·8	+19·3	− 2·3	−34·6	+29·9

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TABLE XI. (x.)

Values of M_0.

	6·0.	7·5.	8·5.	9·0.	9·4.	Mean Gradient.
Auwers	+46·7	+35·9	+19·1	0·0	−20·0	+18·9
Table X. (x) Means	+69·1	+55·1	+21·6	0·0	−32·3	+29·9

TABLE XII. (x).

Systematic P.M.'s in R.A. deduced from the $17\frac{1}{2}$ *years' interval between Cambridge Meridian Observations and Oxford Plates, assuming Auwers's determination of the Magnitude Equation to hold good in all R.A.'s.*

Octant of R.A.	Mag. 6·0.		Mag. 7·5.		Mag. 8·5.		Mag. 9·0.		Mag. 9·4.		Mean Grad.	
	New.	Old.	New.	Old.	New.	Old.	New.	Old.	New.	Old.	New. Det^{n}.	Old. Table VII.
h h												
0− 3	+0·7	−6·3	+0·9	−5·5	+0·1	−2·1	−0·5	−2·6	−0·5	+0·9	+0·5	−2·1
3− 6	−0·5	+0·7	−0·9	+0·4	−0·3	+1·7	−0·2	+0·4	−0·8	+0·1	−0·1	−0·1
6− 9	−0·1	+1·4	−0·2	−2·5	−0·1	+0·9	0·0	+0·9	−0·6	+1·9	0·0	−0·8
9−12	−0·5	−2·1	−0·5	+1·6	−0·2	−0·4	−0·5	+0·6	−0·3	−0·1	−0·1	−0·1
12−15	−0·7	−1·1	−0·3	−2·2	−0·2	−0·4	−0·2	−1·0	−1·1	+1·3	0·0	−0·7
15−18	+0·1	+1·5	−0·5	+0·6	−0·4	+0·5	0·0	+1·5	−0·5	−1·9	0·0	+0·5
18−21	−0·1	−1·3	−0·1	−0·8	0·0	−1·4	+0·1	+0·7	−0·4	−0·3	0·0	−0·3
21−24	+0·3	−2·9	+0·1	−0·8	+0·4	+0·7	+0·1	−1·0	−0·5	+0·3	+0·1	−0·8

27. We may now proceed to repeat Table VIII. of the present paper with the new values of μ_α and μ_δ given in Tables XII. (y) and XII. (x).

TABLE XIII.

Corresponding to Table VIII., with the New Values of μ_α *and* μ_δ.

Octant.	μ_α.	sin (α−A).	μ_α/sin (α−A).	μ_δ.	K.	μ_δ/K.	$\pm\mu_\alpha \pm\mu_\delta$. $\pm\sin(\alpha-A)\pm K$.	Gal. Lat.
h h								°
0− 3	+0·5	+·9	+0·6	−0·4	−·6	+0·7	+0·6	−34
3− 6	−0·1	+·4	−0·2	0·0	−·8	0·0	−0·1	−13
6− 9	0·0	−·4	0·0	−0·2	−·8	+0·3	+0·2	+22
9−12	−0·1	−·9	+0·1	−0·1	−·6	+0·2	+0·1	+61
12−15	0·0	−·9	0·0	+0·1	−·3	−0·3	−0·1	+80
15−18	0·0	−·4	0·0	+0·2	(−·1)	(−2·0)	−0·4	+40
18−21	0·0	+·4	0·0	+0·3	(−·1)	(−3·0)	−0·6	+ 2
21−24	+0·1	+·9	+0·1	0·0	−·3	0·0	+0·1	−25

28. The last two columns give the equivalent of Table IX., and we may now compare the values of $\varpi V \cos D \sec \delta$ for different galactic latitudes (as we did in § 14) given by the two methods.

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TABLE XIV.

Octant.	18^h–21^h	3^h–6^h	6^h–9^h	21^h–24^h	0^h–3^h	15^h–18	9^h–12^h	12^h–15^h
Gal. Lat.	+2°	−13°	+22°	−25°	−34°	+40°	+61°	+80°
Old ...	−0·8	−0·9	+0·6	−1·3	−2·3	−0·4	−0·5	+1·0
New ...	−0·6	−0·1	+0·2	+0·1	+0·6	−0·4	+0·1	−0·1

29. It cannot be said that any conclusion emerges at present from such figures; the "new" and the "old" are almost irreconcilable. If the longer interval available for the "new" entitles them to preference, then we must frankly admit that the systematic proper motions found in the former paper (*Monthly Notices*, vol. lxiii. p. 56) and in the early part of this, and any conclusions based on them, are spurious. On the other hand we must remember that the "new" determinations are based on the assumptions—

(*a*) That the magnitude equation is sensibly the same for all R.A.'s.

(*b*) That Auwers's determination of it (in *Ast. Nach.* 3844) is correct.

Neither of these assumptions may be true, but they would have to be seriously wrong to remove the discordances made apparent in Table XIV.

30. It is easy to show the extent to which they must be in error by a graphical method. Let us suppose, as in §§ 17–19, that we are concerned with three definite epochs—1879·0, when the Cambridge observations were made; 1894·0, when the E plates were taken at Oxford; and 1899·0, when the L plates were taken—and for simplicity consider only the quantity which we have called the "mean gradient," which represents the difference *per magnitude* in units of 0″·03.

Let us plot the value of this quantity as ordinate against the date as abscissa. Then, if assumptions (*a*) and (*b*) of the last paragraph are correct, and our determinations of systematic proper motion in the different octants absolutely trustworthy, we should have a diagram like figure (1).

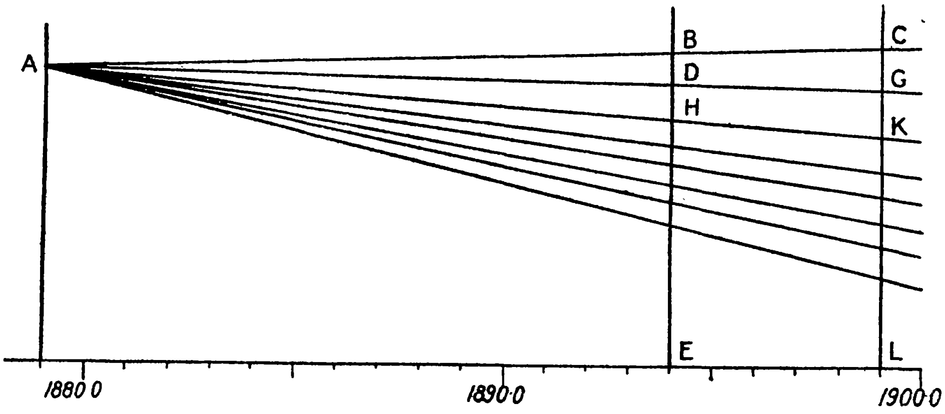

FIG. 1.

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Representing the E value of the mean gradient in any given octant by the point B, and the L value by the point C, then the straight line BC, produced backwards, should pass through the point A, representing Auwers's mean gradient for 1879·0. And for other octants we should have lines ADG, AHK, &c., all passing through the point A.

31 If assumption (*b*) of § 29 is incorrect, but (*a*) still holds, then the various lines would still pass through a point on the line OA, though not the point A. Finally, if assumption (*a*) is incorrect, the eight lines will cut OA in eight different points; but unless the variation of magnitude equation with R.A. is comparable with its mean value, these eight points will be at distances from A small compared with AO.

32. It will be found on consideration of the formation of the last columns of Table X. (*y*) in § 20, and of Table X. (*x*) in § 26, that the quantities therein given represent the values of OA for the different octants. The arithmetical process used corresponds exactly to the production of lines joining points, such as C and B, backwards to cut the line OA.

33. To complete the diagram we want the slopes of the lines; and it is easily seen that these are given by the "old" system of systematic proper motions, as shown in the columns μ_a and μ_δ of Table VIII. in § 11. Hence we construct figures (2) and (3), which correspond to R.A. and declination.

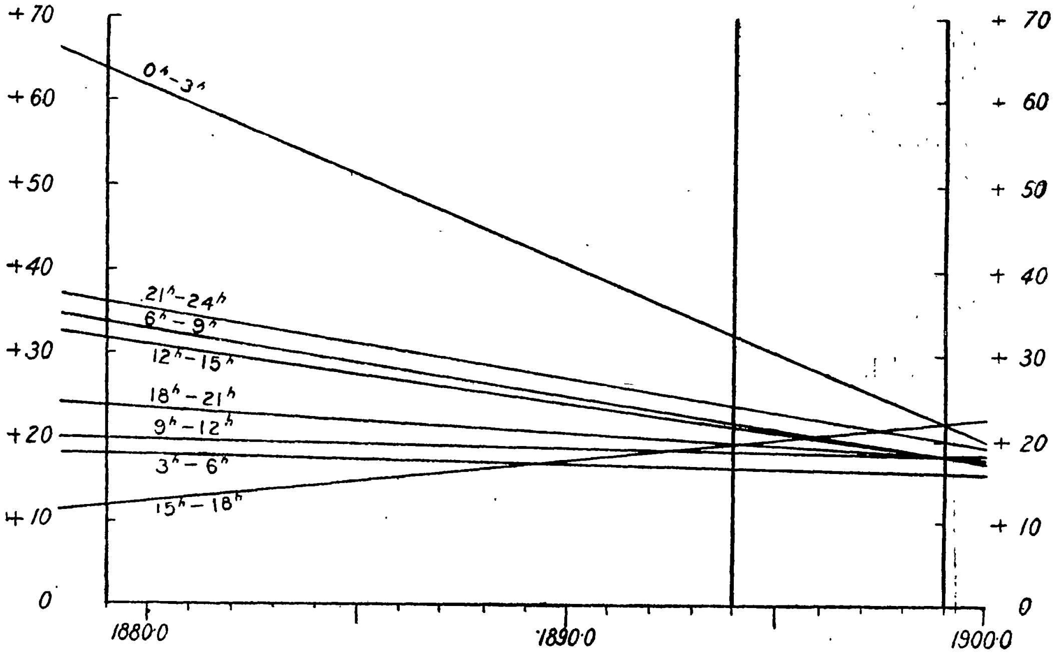

Fig. 2.

The lines are sometimes so close together that it is difficult to distinguish them, and the following small table, which gives the material for constructing the figures, may assist us to follow them. Under OA is given (omitting decimals) the value of the

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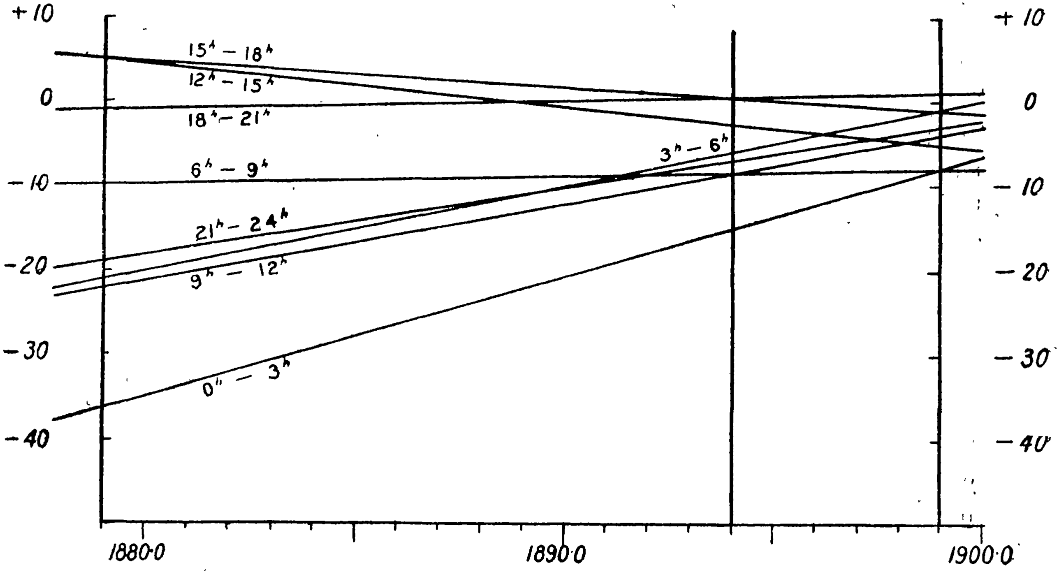

Fig. 3.

mean gradient in 1879·0, as shown in Tables X. (x) and X. (y). Adding twenty years' proper motions, as shown in the columns $20\mu_\alpha$ and $20\mu_\delta$ (got from Table VIII.), we find the values for epoch 1899·0 as shown in the columns headed LC.

Table XV.

Octant.	Fig. (1).			Fig. (2).		
h h	OA.	$20\mu_\alpha$.	LC.	OA.	$20\mu_\delta$.	LC.
0 – 3	+64	−42	+22	−36	+28	−8
3 – 6	+18	− 2	+16	−21	+20	−1
6 – 9	+34	−16	+18	−10	+ 2	−8
9 – 12	+20	− 2	+18	−22	+18	−4
12 – 15	+32	−14	+18	+ 5	−10	−5
15 – 18	+12	+10	+22	+ 5	− 6	−1
18 – 21	+24	− 6	+18	− 1	+ 2	+1
21 – 24	+36	−16	+20	−19	+16	−3

34. It is apparent either from the figures or from Table XV. that the lines do not tend to converge to a point on the line for 1879·0. There is rather an apparent tendency to converge to a point on the line for 1899·0; but this is somewhat spurious. Producing the lines 20 years into the future it will be seen that the convergence is really to the epoch 1894–1899 generally, and is a natural effect of the admixture of "accidental" errors of some kind. These "accidental" errors may quite reasonably be assigned to real individual proper motions of the stars; but it is of course a possibility, which must be faced, that they are due to errors of measurement. Against this latter alternative we may,

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however, set the number of plates measured, and the accordance shown by different zones in the former paper (§ 9, *Monthly Notices*, vol. lxiii. p. 60).

35. But it is clearly better to show at once the nature of the evidence previously collected; and I should here acknowledge that Professor Seeliger has pointed out (*Ast. Nach.* 3865) from independent considerations that there must be a large element of accidental error in the results given in the former paper.

Summary.

36. (*a*) In a previous paper (*Monthly Notices*, vol. lxiii. p. 56) the relative systematic proper motions in R.A. of bright and faint stars in an interval of five years, as found from photographic plates taken at Oxford, were found by an indirect method, using Cambridge meridian observations made twenty years earlier as an intermediary.

(*b*) In the present paper the same method is extended to proper motions in declination.

(*c*) Attributing the proper motions so deduced to the parallactic effect of the Sun's motion in space, and determining thence the relative parallax of bright and faint stars in different octants of the R.A. circuit from the two components, a fair agreement was obtained, which suggested some reality for the determination (see § 15).

(*d*) But on using a different method for testing the results obtained they were not confirmed. This different method depends on one or more assumptions (see § 29), but it is not likely that these assumptions are so incorrect that the former method can still be accepted.

(*e*) It must, therefore, be admitted that either the interval of five years is not sufficiently long, in spite of the number of stars measured, to enable these relative proper motions to be determined, or that the systematic proper motions are so small compared with intrinsic proper motions that we require a much larger number of stars to get trustworthy mean values.

Errata in former paper (*Monthly Notices LXIII., pp.* 61, *&c.*).

Page 61. Last line but two: col. 2 *for* −0·9 *read* −1·3; col. 7 *for* −0·1 *read* −0·3; and col. 8 *for* −0·23 *read* −0·33.
" 61. Last line: col. 2 *for* −1·21 *read* −1·26; col. 7 *for* −0·05 *read* −0·08; and col. 8 *for* −0·55 *read* −0·56.
" 62. Table VIII. first line: *for* −1·21 *read* −1·26; *for* +0·44 *read* +0·39.
" 68. Line 13 ($18^{h}-21^{h}$) *for* −0·2 *read* −0·8; *for* −0·6 *read* −0·8.
" 68 Line 20: *for* −0·6 *read* −0·8.
" 68. Last line but three: *for* $a-2m$ *read* $a-{\scriptstyle 2m}$.